①

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2(x+h)^2 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{(x^2+2xh+h^2)(x+h)^2 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{x^2(x+h)^2 + 2xh(x+h)^2 + h^2(x+h)^2 - x^4}{h}$$

* factor out h *

$$= \lim_{h \to 0} \frac{\cancel{x^4} + 2x^3h + x^2h^2 + 2xh(x+h)^2 + h^2(x+h)^2 - \cancel{x^4}}{h}$$

$$= \lim_{h \to 0} 2x^3 + x^2h + 2x(x+h)^2 + h(x+h)^2$$

$$= 2x^3 + 0 + 2x(x+0)^2 + 0$$

$$= 2x^3 + 2x^3$$

$$= 4x^3.$$

(2) $$\frac{d}{dt}\left(t^2 e^{-ct}\right) = 2t\,e^{-ct} + t^2 e^{-ct}\cdot(-c)$$

$$= 2te^{-ct} - ct^2 e^{-ct}$$

(3) $$\frac{d}{dx}\left(\frac{x^2+\sqrt{x}+1}{x^{3/2}}\right) = \frac{d}{dx}\left(x^{2\cdot\left(-\frac{3}{2}\right)} + x^{\frac{1}{2}}x^{-\frac{3}{2}} + x^{-\frac{3}{2}}\right)$$

$$= \frac{d}{dx}\left(x^{-3} + x^{-3/4} + x^{-3/2}\right)$$

$$= -3x^{-4} - \frac{3}{4}x^{-7/4} - \frac{3}{2}x^{-5/2}$$

(4) $$\frac{d}{dx}\left(\frac{x^3}{9}\ln(3x-1)\right) =$$

$$\frac{3x^2}{9}\cdot\ln(3x-1) + \frac{x^3}{9}\,\frac{1}{3x-1}\cdot 3 =$$

$$\frac{x^2}{3}\left(\ln(3x-1) + \frac{x}{3x-1}\right)$$

(5) $$\frac{d}{dx}\left((3x^2+5)^3(3x^3-2)^2\right) =$$

$$3(3x^2+5)^2(6x)(3x^3-2)^2 +$$

$$(3x^2+5)^3 \cdot 2 \cdot (3x^3-2) \cdot 9x^2 =$$

$$18x\,(3x^2+5)^2(3x^3-2)^2 +$$

$$18x^2(3x^2+5)^3(3x^3-2) =$$

$$18x\,(3x^2+5)^2(3x^3-2)\left(3x^3-2+3x^2+5\right) =$$

$$54x(3x^2+5)^2(3x^3-2)(x^3+x^2+1)$$

⑥ $$\frac{d}{dx}\left(x\ln(x)-x+2\right) =$$

$$\ln(x)+\frac{x}{x}-1 = \ln(x)$$

⑦ $$\frac{d}{dx}\left(\sqrt{\frac{x^2+9}{x+3}}\right) = \frac{1}{2}\left(\frac{x^2+9}{x+3}\right)^{-\frac{1}{2}}\frac{d}{dx}\left(\frac{x^2+9}{x+3}\right)$$

$$= \frac{1}{2}\left(\frac{x^2+9}{x+3}\right)^{-\frac{1}{2}}\left(\frac{2x(x+3)-(x^2+9)\cdot 1}{(x+3)^2}\right)$$

$$= \frac{1}{2}\left(\frac{x^2+9}{x+3}\right)^{-\frac{1}{2}}\left(\frac{2x^2+6x-x^2-9}{(x+3)^2}\right)$$

$$= \frac{1}{2}\left(\frac{x^2+9}{x+3}\right)^{-\frac{1}{2}}\left(\frac{x^2+6x-9}{(x+3)^2}\right)$$

8) $\frac{d}{dt}\left(6t^{-2} + 3t^{3} - 4t^{\frac{1}{2}}\right)$

$= -12t^{-3} + 9t^{2} - 2t^{-\frac{1}{2}}$

$\frac{d}{dt}\left(-12t^{-3} + 9t^{2} - 2t^{-\frac{1}{2}}\right) =$

$36t^{-4} + 18t + t^{-3/2}$